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This grant has the central aim of exploring when, and how, probability is useful in the theory of algorithms. Most of the problems reviewed have their origins in the area of Euclidean Combinatorial Optimization, which might be operationally defined as the theory that has evolved out of the study Euclidean traveling salesman problem (TSP), the minimal spanning tree problem, and the minimal matching problem. Probability enters the study of such problems by two different paths. One path calls on exogenous randomization in the course of a genuine probabilistic algorithm. This path is of increasing importance in many areas, and on an elementary level is well illustrated by the method of simulated annealing. A second path of considerable importance calls on the introduction of stochastic models for the problem inputs. One then uses probability theory to understand as deeply as possible the behavior of the associated objective functions. This understanding is used subsequently to guide algorithm design.

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Probability and Statistics Applied to the Theory of Algorithms

FINAL REPORT

Grant AFOSR-91-0259

University of Pennsylvania

(a transfer of AFOSR-89-0301 from Princeton University)

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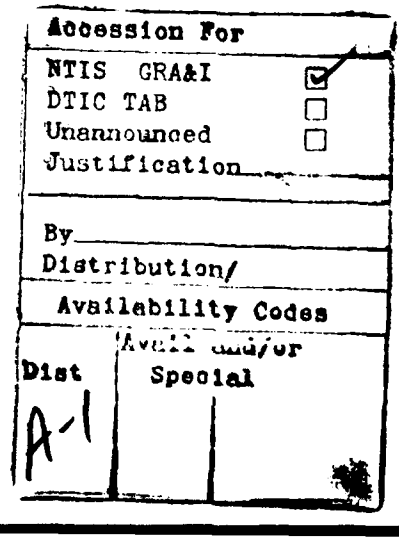
December, 1992



1 Orientation

This report principally focuses on the progress that has been made with the support of AFOSR-91-0259 at University of Pennsylvania, but, since this grant originated as a transfer of grant AFOSR-89-0301 from Princeton University the work completed with that support is also briefly reviewed. The next section provides a top-down view of the issues that have been investigated with this support, and then in Section Three there is a summary of the major contributions. Section Four provides a list of the articles supported by AFOSR-91-0259, and gives a citation to the authors publication list for additional work supported by AFOSR-89-0301. A brief description of the principal investigators other grant related professional activities is given in Section Five. Next, Section Six provides a list of scientific personnel who have helped with this work and received some support from the research funds. Additionally, Section Six discusses the two doctoral dissertations completed with help from this grant and the additional dissertation work that is still underway. The final section aims to suggest further activities that tie into the work that has been completed.

2 Nature of the Problems Studied

This grant has the central aim of exploring when, and how, probability is useful in the theory of algorithms. Most of the problems reviewed here have their origins in the area of Euclidean Combinatorial Optimization, which might be operationally defined as the theory that has evolved out of the study Euclidean traveling salesman problem (TSP), the minimal spanning tree problem, and the minimal matching problem. Probability enters the study of such problems by two different paths. One path calls on exogenous randomization in the course of a genuine probabilistic algorithm. This path is of increasing importance in many areas, and on an elementary level is well illustrated by the method of simulated annealing. A second path of considerable importance calls on the introduction of stochastic models for the problem inputs. One then uses probability theory to understand as deeply as possible the behavior of the associated objective functions. This understanding is used subsequently to guide algorithm design. This second path is well-illustrated by Karp's algorithm for the TSP.

3 Summary of Major Contributions

The Objective Method. Probably the best work initiated with the assistance of this grant is represented by the article "Asymptotics of Euclidean Minimal Spanning Trees on Random Samples," written with David Aldous and published in *Probability and Related Fields.* This article resolves an interesting conjecture of R. Bland's that the sum of the squares of the edges of the minimal spanning tree of a random sample from the unit square converges to a constant as the sample size goes to infinity. Although it is reinforcing to have resolved Bland's conjecture, the main contribution of the article is to provide a compelling example of the "objective method" whereby one obtains limit results by constructing an "infinite analog of the object of interest." The justification that one has the right infinite analog then requires showing that ones candidate provides concrete insight into the large sample behavior of the original finite problem. In the problem addressed with Aldous, both parts of the program were brought to a satisfying conclusion.

This is a rich method that continues to make contributions to other problems. Aldous and I have found several additional examples of the method that show its potential and demonstrate that the method deserves to be distinguished from the methods of weak convergence, or invariance principles. There are links between these methods and the "objective method," but there are also distinctions that merit being preserved. Aldous and I continue to work independently and jointly on the development of this area.

Symbolic Methods and Itô Calculus. The article "Applications of **Mathematica** to Stochastic Calculus" written with R. Stine reports on an implementation of the basic formalism that arises from Itô's formula in the **Mathematica** platform. The tool that results greatly shortens many of the routine calculations of the stochastic calculus. The article is one that has created considerable interest and there have been numerous request for the code that we have developed. This work began casually and it is still not at the center of my research interests, but it has compelling attractions. The first report on this software development was given in the *Proceedings of the ASA Section on Computational Statistics.* A more polished version will appear shortly in a volume published by Springer-Verlag edited by Hal Varian. The volume addresses the uses of **Mathematica** in economics and finance.

Probability and Algorithms: A Panel Report. National Academy of Sciences Board on Mathematical Sciences panel has completed its work, and its report has just been published. I served as the chairman of this panel and contributed to its efforts at each step from project conception (while serving on the BMS Committee on Applied and Theoretical Statistics), to panel selection, through project organization, to final editing. David Aldous and I wrote the introduction to the report; and, in addition to writing a chapter that addresses problems in Euclidean Combinatorial Optimization, I wrote the concluding chapter. The panel report will be reprinted as a special issue of Statistical Science, and I have been active in expediting this reprinting. On the advice of Rob Kass, editor of Statistical Science, I made substantial additions to the introductory chapter, and these additions should serve to make the report more appropriate for the Statistical Science audience.

Other Results. The other contributions reviewed below are more specialized and *a priori* might be thought less likely to be pursued by other researchers. Since the field of Euclidean combinatorial optimization and its interaction with probability is rapidly expanding, this is not necessarily the case. There is an increasing number of vigorous investigators in this new area, and articles that offer progress on good problems can have a substantial impact—both on research methods and on research directions.

1. The article "A priori Bounds for the Euclidean Traveling Salesman Problem" written with T. L. Snyder continues one of my basic lines of research: the development of inequalities for heuristics for NP-hard problems. This article was presented as an invited paper at the *Eighth Symposium in Computational Geometry* in Berlin in June 1992.

2. The article "Convex Hulls of Random Walks" which was also written with T. Snyder provides exact formulae for the length of the convex hull of $\{S_1, S_2, ..., S_n\}$ where $S_k = X_1 + X_2 + \cdots + X_k$ and the X_i are i.i.d. with values in $\Re^2$.

   The results are surprisingly simple and offer an instance of combinatorial and geometric methods coming together in succinct and definitive results. The work was motivated by earlier contributions by Spitzer and Baxter. The article has been accepted for the *Proceedings of the American Mathematical Society.*

3. The article "Transient Behavior of Coverage Processes" written with S. Browne responded to a target of opportunity. The problem that we solved is that of de-

termining the distribution of the clump size in a coverage process where the first interval in any clump has a distribution that differs from that of the subsequent intervals. The distributional question comes up in a variety of contexts including storage problems, scheduling problems, and queuing problems where the first service of any busy period has a exceptional service length distribution. The article has been accepted by the *Journal of Applied Probability.*

4. The piece " Probabilistic networks and network Algorithms" with T.L. Snyder will appear as part of the North-Holland Series *Handbook of Operations Research.* The article is largely expository, but it also has a research component. In particular it aims to provide a basic grounding suitable for a second-year graduate student interested in beginning research in the material of this grant.

4 Research Articles Supported by this Grant

1. "Asymptotics of Euclidean Minimal Spanning Trees on Random Samples," (with David Aldous), *Probability Theory and Related Fields* **92**, (1992), 247-258.

2. "*A Priori* Bounds for the Euclidean Traveling Salesman Problem," (with T. L. Snyder), *Proceedings of the Eight Annual Symposium: Computational Geometry*, ACM Press, New York, NY, 1992, 344-349.

3. "Introduction to Probabilistic Algorithms and the Probabilistic Analysis of Algorithms," (with David Aldous). *Probability and Algorithms: A Panel Report*, 1-15, National Academy of Sciences, National Research Council, Board on Mathematical Sciences, Washington D.C.

4. "Probability and Problems in Combinatorial Optimization,". *Probability and Algorithms: A Panel Report,* 109-129, National Academy of Sciences, National Research Council, Board on Mathematical Sciences, Washington D.C.

5. "Missing Pieces, Derandomization, and Concluding Remarks," *Probability and Algorithms: A Panel Report*, 175-178, National Academy of Sciences, National Research Council, Board on Mathematical Sciences, Washington D.C.

6. "Applications of **Mathematica** to Stochastic Calculus" (with R. A. Stine), *ASA 1991 Proceedings of the Statistical Computing Section,* (1991), 11-19.

7. "Transient Behavior of Coverage Processes with Applications to Infinite Server Queues," (with S. Browne), To appear in *Journal of Applied Probability*, September 1993.

8. "Convex Hulls of Random Walks," (with T. L. Snyder). To appear in *Proceedings of the American Mathematics Society.*

9. "Probabilistic Networks and Network Algorithms," (with T. L. Snyder). To appear in *Handbook of Operations Research.*

10. "Asymptotic Equidistribution of Worst-case Point Sets in the Travelling Salesman and Related Problems," (with T. L. Snyder). Accepted for presentation at STOC '92 in Austin 1992, an extended version of the article has also been submitted to **SIAM J. Computation.**

11. "General Spacefilling Curve Heuristics and Limit Theory for the Traveling Salesman Problem," (with J. Gao), to appear in the *Journal of Complexity.*

Additional Related Publications. In addition to these articles, the items numbered 11,13,15,18,19,20,21,74, and 77 of the attached list of publications either acknowledge the support of this grant, or the earlier version AFOSR-89-0301, or are directly related to this work.

5 Grant Related Professional Activities

National Academy of Sciences: Board on Mathematical Sciences. Committee Report on Cross-Disciplinary Research. I recently completed service as chair of a NAS-BMS Committee charged with developing a report "Probability and the Theory of Algorithms." The panel report has been published by the NRC, and it will be reprinted in a forthcoming issue of *Statistical Science.*

Institute of Mathematics and Its Applications: Special Year in Applied Probability. I currently serve as chair of the IMA Organizing Committee for the Special Year in Applied Probability to be held in 1993-4. The year's activities will include about ten Workshops in Applied Probability, six to ten post-doctoral visitors to the IMA, about six long-term senior visitors, and several dozen short-term visitors. The year long activity may well provide the biggest single push that has been experience in the field of Applied Probability. It offers the opportunity to have an impact that will be felt for many years.

Annals of Applied Probability I currently serve as Editor of *The Annals of Applied Probability*, the most recently created journal of the Institute of Mathematical Statistics. After years of review and discussion, the IMS acted to form this addition to the successful *Annals of Probability* and *Annals of Statistics.* The new journal is off to a solid start, and, after two years of publication, now receives a steady flow of quality submissions. There are good reasons to regard the new *Annals* as the leading journal in its field.

6 Participating Scientific Personnel and Advanced Degrees Earned

1. **J. Michael Steele** (Principal Investigator)

2. **T. L. Snyder** (Associate Professor of Computer Science, Georgetown University). Dr. Snyder received some assitance during summer visits to the University of Pennsylvania. He has contributed as a co-author on several of the referenced articles.

3. **J. Gao.** Mr. Gao began his graduate research while I was at Princeton University, and has received some support from the present grant while completing his thesis. Gao finished the work for his Princeton Ph.D. his dissertation "Analysis of Two Heuristic Methods for the Euclidean Traveling Salesman Problem" in September, 1992. One of the articles that originated in this work "General Spacefilling Curve Heuristics and Limit Theory for the Traveling Salesman Problem," is to be published in the *Journal of Complexity.*

4. **Nanda Piersma.** Ms. Piersma is a fourth year graduate student at the Tinbergen Institute in Amsterdam. She has visited the University of Pennsylvania for two summers and one academic year. I am serving as her dissertation advisor, and I expect her to finish her degree by June 1993. The area of her dissertation research is closely connected to the problems of this grant. It is likely that her thesis will lead to several publications that will acknowledge this support.

5. **Makki Momma.** Mrs. Momma completed her dissertation at Princeton University in October 1990. Her work has focused on the distribution and sampling theory of intercepted samples. The simplest example of such sampling consists of looking at the age and eventual lifetime of (say) an electronic component that is currently in use, or of a computer job that is currently running. The fact that such an "intercepted" item is still alive prevents it from being used naively to obtain an unbiased estimate of the lifetime distribution (of an item from the population of all items that have been placed in service). This observation leads to a theory of length biased sampling distributions that offers many applications and insights. This theory has been developed most forcefully by Y. Vardi (formerly of Bell Laboratories, currently at Rutgers).

   The heart of our understanding the performance of many heuristic methods comes via the interpretation of simulations, and one of the important issues being addressed by Mrs. Momma is the application of biased sampling models to simulations. The results are not yet definitive, but biased sampling model seems to contribute a deeper understanding of real-time routing in communication networks and of other real-time heuristics like those employed in the self- organizing list.

6. **Claude Athaide.** Mr. Athaide is currently a fourth-year graduate student in Statistics at the University of Pennsylvania. His thesis work concerns the uses of geometric combinatorial optimization in problems of classification, recognition, and clustering. These problems are on the most practical frontier of the field addressed by this grant, and his work aims to develop statistical applications of recent computational advances. He is also engaging work in non-linear filtering.

7 Concluding Remark

The work that has been surveyed here continues to be at the center of my research interest. During my leave next year at the IMA, I hope to make progress in the areas that have been opened by the work reported here. The benefits that probability theory offers to the theory of algorithms today stand as central to several more specialized areas. Moreover, the contributions that one now expects from the applications of probability in computer science are many fold more substantial than anyone would have suspected ten year ago, so the present lines of investigation should continue to deserve the attention of the AFOSR and of the broadest research community.

RESEARCH PUBLICATIONS

November 1992

J. Michael Steele

Published Articles:

[1] "Asymptotics of Euclidean Minimal Spanning Trees on Random Samples," (with David Aldous). *Probability Theory and Related Fields* **92**, (1992), 247-258.

[2] "*A Priori* Bounds for the Euclidean Traveling Salesman Problem," (with T. L. Snyder), *Proceedings of the Eight Annual Symposium: Computational Geometry*, ACM Press, New York, NY, 1992, 344-349.

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[5] "Probability and Problems in Combinatorial Optimization," *Probability and Algorithms: A Panel Report*, National Academy of Sciences, National Research Council, Board on Mathematical Sciences, 1992, Washington D.C., 109-129.

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[73] "Transient Behavior of Coverage Processes with Applications to Infinite Server Queues." (with S. Browne), To appear in *Journal of Applied Probability*, September 1993.

[74] "LeCam's inequality and Poisson approximation," To appear in *The American Mathematical Monthly.*

[75] "Convex Hulls of Random Walks," (with T. L. Snyder). To appear in *Proceedings of the American Mathematics Society.*

[76] "Probabilistic Networks and Network Algorithms," (with T. L. Snyder). To appear in *Handbook of Operations Research*

[77] "General Spacefilling Curve Heuristics and Limit Theory for the Traveling Salesman Problem," (with J. Gao). To appear in *Journal of Complexity* and to be reprinted in *The Festschrift for Joseph Traub.*

[78] "Mathematica and Diffusions" (with R. A. Stine). To appear in *Symbolic Computation in Economics and Finance*, Hal Varian, editor, Springer-Verlag, New York.

Articles Under Submission and Working Papers Available for Comment:

[79] "Sums of Squares of Edge Lengths and Spacefilling Curve Heuristics for the Traveling Salesman Problem," (with J. Gao). Submitted to *SIAM J. Computation.*

[80] "Asymptotic Equidistribution of Worst-case Point Sets in the Travelling Salesman and Related Problems," (with T. L. Snyder). Submitted to *SIAM J. Computation.*

[81] "Complexity-Based Tools for Automated Learning," (with T. L. Snyder).

[82] "Uniform Strong Laws of Random Lines," (with N. Piersma).

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[83] Review of *Theoretical and Computational Aspects of Simulated Annealing* by P. J. M. van Laarhoven (CWI Tract, Centrum voor Wiskunde en Informatica, Amsterdam), *J. Amer. Statists. Soc.*,**85**,(1990), 596-596.

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[88] "Red flags on the Houston study," SIMS Working Paper No. 14, Stanford University.

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